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IMAGE PROCESSING-BASED DETECTION OF OBSERVABLE FARADAY ROTATION IN COHERENT SUNLIGHT

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ABSTRACT

This work presents a novel demonstration of the optical Faraday effect using spatially coherent sunlight, providing an alternative to traditional laser-based approaches. The beam was directed through a Terbium Gallium Garnet (TGG) magneto-optic crystal under a tunable magnetic field (-0.5 to 0.5 T). The polarization rotation was visualized through a linear polarizer that produced a split-lobe intensity pattern whose angular shift was directly related to the applied magnetic field. Experimental results showed a linear dependence of the rotation angle on the magnetic field magnitude and were in close agreement with theoretical predictions. This work validates the feasibility of utilizing natural sunlight for high-precision magneto-optic experiments, overcoming the limitations associated with artificial coherent sources. The methodology not only advances the fundamental understanding of light-matter interactions but also highlights practical applications in developing field-deployable optical devices such as magnetic field sensors and sunlight-powered isolators. By combining classical magneto-optic principles with innovative imaging techniques, this research opens avenues for sustainable, cost-effective optical technologies that utilize natural light sources.

Keywords: Faraday effect, OAM, White light, Optical vortex.

1 INTRODUCTION

The optical Faraday effect, also known as Faraday rotation, describes and indicates the rotation of the polarization plane of light that passes [1] through a magneto-optic material when

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subjected to a magnetic field aligned with the direction of propagation. First reported by Michael Faraday in 1845, this phenomenon marked the first experimental evidence of the intrinsic relationship between light and magnetism and provided the basis for later theoretical developments in electromagnetism [1-3].

Quantitatively, the angular rotation θ is given by $\theta = VBL$, where V denotes the Verdet constant (wavelength- and material-dependent), B the magnetic flux density, and L the interaction path length. The Verdet constant is significant for evaluating material performance in Faraday devices [3-5]. Materials such as terbium gallium garnet (TGG) and other paramagnetic crystals typically exhibit high Verdet values across visible and near-infrared wavelengths. Recent material classes such as ceramics, nanocomposites, and fluoride-based crystals are being explored to obtain both higher Verdet constants and reduced thermal effects under high-power applications [6].

The Faraday effect has long played an important role in both fundamental and applied optics. It serves as a sensitive probe in magnetization dynamics and spintronics, while in applied physics it underpins devices like optical isolators and Faraday rotators, essential components in laser systems for preventing back-reflections and ensuring directional stability. The demand for robust magneto-optic components in high-power lasers further motivates ongoing material development and thermal management strategies.

Experimental studies traditionally rely on coherent laser sources due to their stable polarization, narrow linewidth, and ease of modulation [6-11]. Nevertheless, the potential of natural spatially coherent sunlight remains largely untapped. Utilizing sunlight with polarization structuring offers unique possibilities for demonstrating Faraday rotation in accessible, non-lab environments and for developing optical systems that are both cost-effective and suitable for field deployment.

Recent literature highlights ground-breaking advances in high-Verdet materials. For instance, two-dimensional transition metal dichalcogenide monolayers like WSe_2 and $MoSe_2$ encapsulated in hBN exhibit extraordinarily large Verdet constants on the order of 10^9 degree/ T m which is far exceeding traditional bulk crystals in the visible regime. Such giant Faraday rotation promises ultrahigh-performance magneto-optic modulators and sensors at the atomic scale [12].

Another emerging research direction is the exploration of nonlinear Faraday effects in fiber optics. Recent theoretical work on helically wound nonlinear optical fibers demonstrates

additional rotation contributions that depend on optical intensity as well as magnetic field suggesting that Faraday rotation can be fine-tuned via optical power in addition to conventional parameters [13].

Magneto-optical thin films and integrated photonic devices are gaining importance for practical applications. Some reports advance in these materials and their use in isolators, modulators, sensors, and magnetometers, showing their value for small, chip-based optical systems [14].

In experiments, it is very important to measure the Verdet constant accurately for different materials, wavelengths, and temperatures. A recent review explains methods for this measurement, compares materials for use in high-power Faraday devices, and shows that not only a high Verdet constant but also good optical transparency, strong heat conduction, and low birefringence are necessary for the best performance [15].

This research not only extends the understanding of the optical Faraday effect using natural light sources but also paves the way for innovative applications in real-world optical systems. The findings highlight the feasibility of utilizing coherent sunlight in advanced optical experiments and offer valuable insights into the dynamic interaction between light polarization and magnetic fields.

2 MATERIAL AND METHOD

Here, an innovative experimental approach to demonstrating the optical Faraday effect using spatially coherent sunlight is presented. In this method, a zero-order vortex half-wave retarder is utilized to produce a doughnut-shaped beam with radial polarization. After passing through the retarder, the sunlight is directed through a linear polarizer, creating two distinct intensity lobes. The gap between these lobes aligns parallel to the polarization direction. The setup includes a magneto-optics material placed in a systematically varied magnetic field, through which the light travels before encountering a final linear polarizer. This results in an observable rotation of the lobe pattern. This phenomenon allows for direct observation of the polarization changes induced by the magneto-optic crystal [16].

In the experiments, unfiltered broadband sunlight was used directly as the light source. No additional spectral filters were employed, as the focus of the study was on the polarization rotation rather than wavelength-dependent intensity variations.

In our setup, coherent sunlight is precisely aligned and focused through a series of optical lenses to pass through a high Verdet constant magneto-optical crystal [12,13]. The crystal is strategically positioned within an adjustable magnetic field generated by electromagnets. Polarizers placed before and after the crystal determine the beam's initial and altered polarization states, respectively. A camera captures the intensity profiles of the sunlight beam at various magnetic field strengths.

The experimental procedure begins with a calibration phase, capturing baseline intensity profiles without any magnetic field. Subsequently, the magnetic field strength is incrementally increased, and corresponding images are recorded. These images are processed to extract the intensity distribution and analyze the rotation degree of the polarization plane. By comparing the intensity profiles under different magnetic field conditions to the baseline, we quantify the Faraday rotation angle using the relationship:

$$\Theta = VB_0L \tag{1}$$

where Θ is the rotation angle, V is the Verdet constant of the crystal, B_0 is the magnetic field strength, and L is the length of the crystal [14]. Comprehensive error analysis accounts for uncertainties in magnetic field measurements, image resolution, and potential misalignments in the setup. The experimental results are then compared with theoretical predictions to verify the accuracy and reproducibility of the measurements.

Radially polarized Laguerre-Gaussian (LG) laser mode can be written as a symmetric combination of two opposite circularly polarized LG modes such that:

$$E = \frac{u(\rho)}{\sqrt{2}}\left\{e^{-i\phi}(\hat{x} + i\hat{y})e^{ikzn^{(+)}} + e^{i\phi}(\hat{x} - i\hat{y})e^{ikzn^{(-)}}\right\} \tag{2}$$

where $u(\rho)$ is the amplitude function of the LG mode winding number $l = 1;\ (p = 0)$, which reads

$$u(\rho) = e^{-\frac{\rho^2}{\omega_0^2}}\left(\frac{\sqrt{2}\rho}{\omega_0}\right)L_0^1\left(\frac{2\rho^2}{\omega_0^2}\right) \tag{3}$$

Here, k is the axial wavevector, n^+ and n^- are the refractive indices of the magneto-optic medium for left-hand circularly-polarised light (+) and right-hand circularly-polarised light (-), respectively. In the case of

$$n^{(+)} = n^{(-)} = 1 \tag{4}$$

Equation (2) will be converted to

$$E = u(\rho)\hat{\rho} \tag{5}$$

where $\hat{\rho}$ is the radial unit vector. And this can be written more explicitly as follows

$$E = \sqrt{2}u(\rho)e^{ikLn_0}\{\cos(\phi - \Theta)\,\hat{x} + \sin(\phi - \Theta)\,\hat{y}\} \tag{6}$$

which is the electric field of the light beam after traversing the length L of the magneto-optic medium under the applied magnetic field. For a given material with a known Verdet constant V and a fixed length L, the Faraday rotation angle Θ can be modified by a change in the applied axial magnetic field B.

A visual means of detecting the angle of rotation can be realized by passing the beam emerging from the magneto-optic medium through a linear polarizer oriented along the x direction, so that the electric field is given by the first term of Equation (6) with the relevant intensity

$$E = u(\rho)e^{ikLn_0}\cos(\phi - \Theta)\,\hat{x} \;\; ; \;\; I = \epsilon_0 c n_0^2 u^2(\rho)\cos^2(\phi - \Theta) \tag{7}$$

so that both the electric field and the intensity are now dependent on both ϕ and Θ.

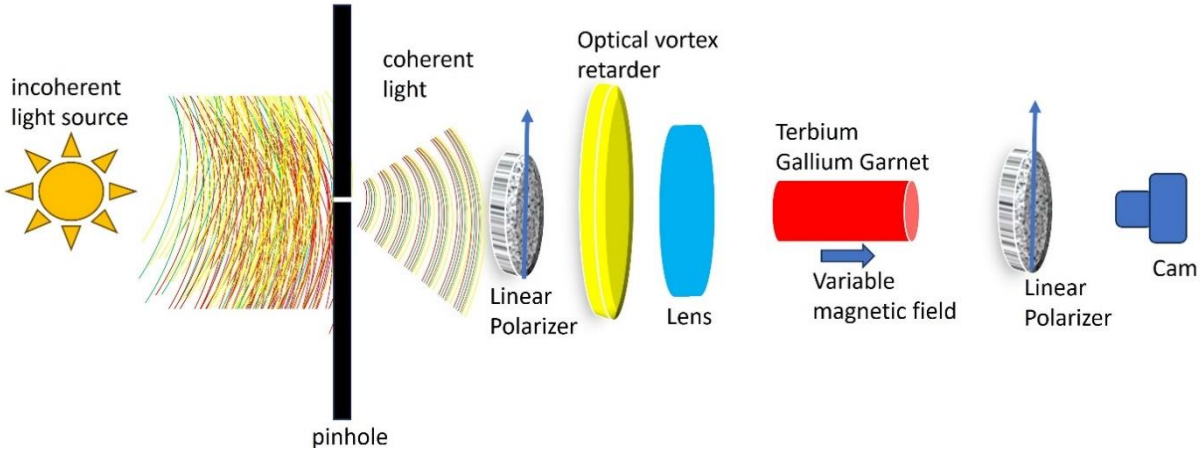


Figure 1. The schematic of the optical setup for transforming incoherent sunlight into a coherent beam using a pinhole.

Our experimental setup is designed to observe the polarization change of spatially coherent and linearly polarized sunlight due to the applied magnetic field on the magneto-optic crystal. As seen in Figure 1, the sunlight has been passed through the pinhole of with a radius of 10 μm to obtain the spatial coherence. Then, the coherent beam has been sent to the linear polarizer to have linear polarization. In the next part of the setup, we have three components which are radial polarization vortex retarder, cylindrical magneto-optic glass rod, and another

linear polarizer. These three components are required for the realization and direct observation of Faraday rotation. In traditional Faraday effect setups, a radial polarization optical vortex retarder is not necessary. Instead of this, one single detector is enough to measure the change in light intensity.

In this study, Terbium Gallium Garnet (TGG) was employed as the magneto-optic material due to its strong frequency-dependent Verdet constant, which has a value of −131 rad/(T·m) at $\lambda = 632$ nm. The magneto-optic element was a cylindrical TGG rod with a length of $L = 10$ mm. An external magnetic field was applied along the direction of light propagation and varied between −0.5 T and 0.5 T. The magnetic field strengths used in the experiments were directly measured with a Hall probe. The intensity of natural sunlight was not measured, since its absolute value is irrelevant for our purpose; what matters is the polarization change induced by the Faraday effect. In fact, regardless of the light intensity, the optical vortex retarder produces a characteristic intensity gap in the beam profile. By applying image processing techniques, the orientation of this gap line was determined for each magnetic field value. A Mathematica code was then used to reproduce the variation of the gap line angle with respect to the horizontal axis. Finally, the experimentally obtained angles were compared with the theoretical predictions.

In our experiment, image processing techniques implemented using Mathematica were meticulously used to analyze polarization changes induced by an external magnetic field on a magneto-optic crystal. Initially, sunlight was filtered through a pinhole and a linear polarizer and then passed through an optical vortex retarder. This configuration resulted in a unique polarization pattern that was further manipulated by the magneto-optic crystal. The magneto-optic effect caused the polarization state to change in response to changes in the external magnetic field. As a result, after passing through a second linear polarizer, the light formed a circular light spot with a distinct dark line bisecting it, which served as an indicator of the polarization changes.

Advanced image processing techniques in Mathematica were used to analyze the recorded images of the light spot. In particular, the dark line dividing the circular spot in two provided important information about the direction of polarization. By accurately measuring the angle of this line relative to the horizontal axis, changes in the polarization state could be observed. The precision of these measurements was crucial because the position and orientation of the dark line are directly affected by the external magnetic field applied to the magneto-optic

crystal. Therefore, each image captured under different magnetic field strengths provided a data point for understanding the relationship between field strength and polarization angle.

Through this method, we successfully mapped the angular changes in polarization as a function of the applied magnetic field. Utilizing Mathematica's powerful image processing capabilities allowed for precise detection of subtle changes in the angle of the dark line, offering a visual and quantitative analysis of the magneto-optic effect.

3 RESULT AND DISCUSSION



Figure 2. Some of the captured raw images of the sunlight spot after passing through the optical setup under different magnetic field strengths. In the five figures, the magnetic field increases progressively from left to right (0, 0.125, 0.25, 0.375, and 0.5 T), with the leftmost image corresponding to zero field and the rightmost to the maximum.



Figure 3. Binary-converted images obtained by applying a threshold to isolate the main intensity lobes.



Figure 4. Processed images highlighting the dark gap region introduced by the optical vortex retarder.

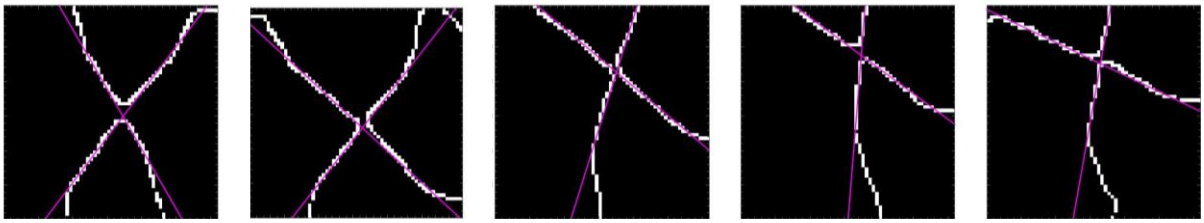


Figure 5. Extraction of the gap line by image processing; fitted curves are superimposed on the processed images.

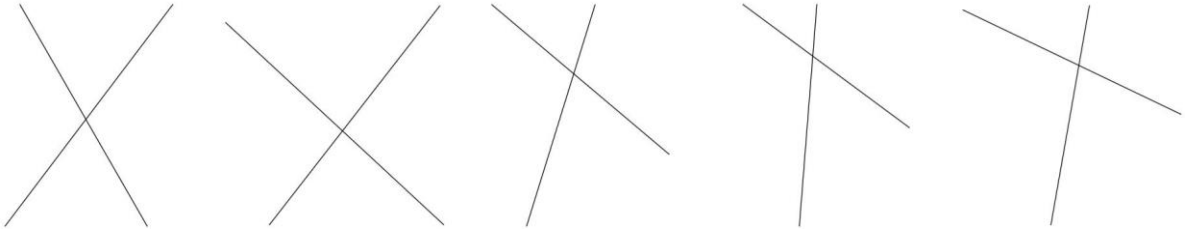


Figure 6. Final determination of the splitting direction, where the fitted lines are represented as straight lines, and the corresponding angles with respect to the horizontal axis are obtained.

Figure 2-6 shows the image processing pipeline used to analyze the rotation of the light spot axis under varying magnetic field strengths. Figure 2 shows the raw light spots captured under different magnetic field conditions. These images show the initial state of the light spots, where the polarization and subsequent light distribution are affected by the applied magnetic fields. The figure 3 depicts the result after applying the Binarize function, which converts the original grayscale images to binary format. This step is important to isolate the light spot from the background and make it easier to identify the area of interest for further analysis. Binary images clearly outline the light spots and provide a clear contrast between the light spot and the surrounding area. The figure 4 shows the application of the EdgeDetect function, which highlights the boundaries of the light spots. This function is crucial to precisely identify the edges of the spot, which is necessary to analyze the rotation axis. By highlighting the edges, changes in the orientation of the light spots under different magnetic field strengths can be measured more accurately. In figure 5, the HighlightImage function is used to superimpose the detected edges onto the original images, allowing for clearer visualization of axis orientation changes. In addition, lines representing the main axes of the light spots are drawn to more clearly show the rotation. These lines are calculated based on the orientation of the detected edges and provide a quantitative measure of the rotation caused by the magnetic fields. Figure 6 presents the calculated axis lines in isolation, highlighting the angular displacement observed in each case. The systematic rotation of the axis as the magnetic field changes is visually

apparent, confirming the effect of the magnetic field on the polarization and orientation of the light spot. This comprehensive image processing approach effectively demonstrates the relationship between the magnetic field strength and the rotation of the polarization axis, providing valuable data for further analysis and interpretation. It is known that the polarization of light is affected by magnetic fields, which in turn affects the orientation of the axis of the light spot as seen in the captured images. To measure and analyze this phenomenon, images were recorded under varying magnetic field strengths. These images were then processed using Mathematica to determine the exact rotation of the axis.

To extract relevant information from the images, several image processing techniques were employed. Initially, the ImageTrim function was used to remove unnecessary parts of the image, focusing solely on the region of interest where the light spot appeared. Following this, ImageCrop was applied to standardize the dimensions of the images, ensuring consistent analysis across different magnetic field strengths. The images were then converted to binary form using the Binarize function, which simplified the identification of the light spot by distinguishing it from the background and which automatically determines the threshold value from the image histogram. This step was crucial for isolating the spot and preparing it for further analysis.

Next, the EdgeDetect function was used to highlight the boundaries of the light spot, making it easier to identify any rotation or shift in the axis. To visualize these changes more effectively, the HighlightImage function was implemented, which marks the detected edges and highlights changes in the axis orientation. Thanks to this sequence of image processing steps, the degree of axis rotation could be accurately determined for each magnetic field strength, providing insights into the relationship between magnetic fields and light polarization.

In Figure 7, each line corresponds to the perceived orientation of a point of light and shows the effect of the magnetic field on the polarization direction. The clustering of lines indicates the coherent rotation caused by increasing magnetic field strength.

Here, we investigated the effect of magnetic fields on the polarization of light, which results in a perceptible rotation of the axis of the light spot in the captured images. The process began with capturing light spots at different magnetic field strengths. These images went through a series of image processing steps, including binarization and edge detection to isolate and highlight the edges of the light spots. This allowed us to precisely determine the orientation of the axes of the spots.

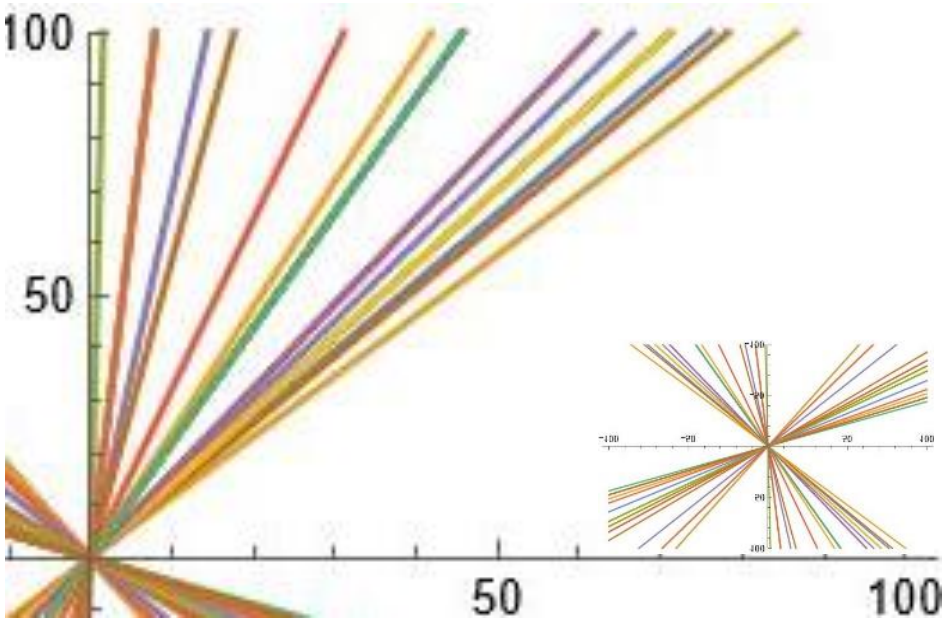


Figure 7. Calculated lines representing the orientation of the splitting axes of the light spots under varying magnetic fields.

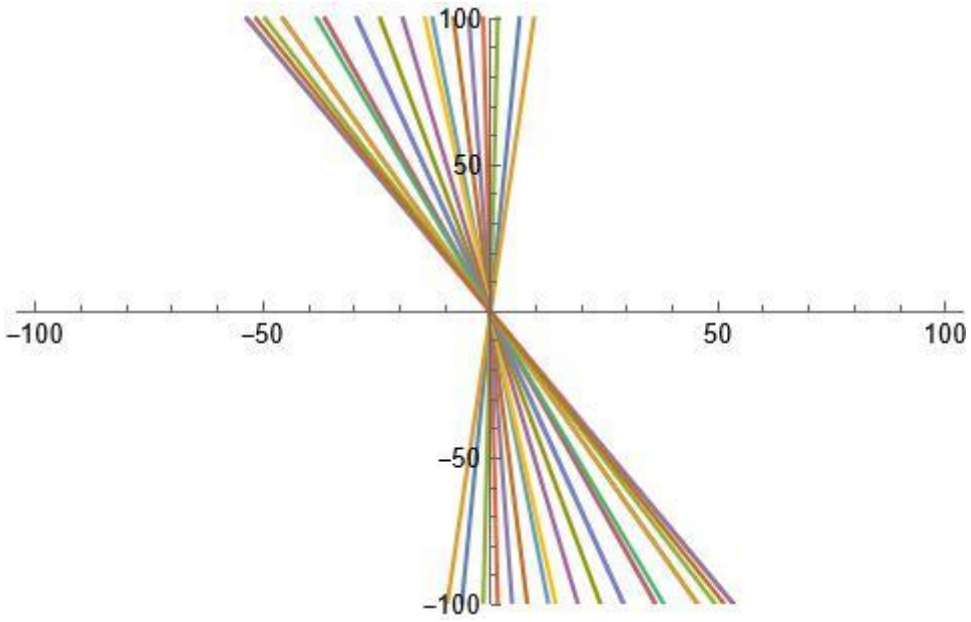


Figure 8. Illustration of the bisectors calculated between each pair of light spot axes, highlighting the median axes.

The bisectors, in figure 8 shown as additional lines, provide a clear representation of the average rotation angle in response to magnetic fields. The angles between these bisectors and the x-axis quantify the Faraday rotation effect, showcasing the linear relationship between magnetic field strength and optical axis rotation. To quantify the axis rotation, we calculated the mathematical equations of the lines representing the main axes of the light spots. By

comparing these lines, we determined the bisector between each pair of lines, which served as the median axis. The angle between this median axis and the x-axis was then calculated, providing a measure of the rotation angle caused by the magnetic field.

The resulting figures display the lines representing the axes of the light spots for various magnetic field strengths, with each line corresponding to a specific field strength. The figure 4 shows the initial angles, while the figure 8 illustrates the additional lines obtained by calculating the bisectors and their corresponding angles. This comprehensive approach allowed us to determine the Faraday angle for each magnetic field strength, demonstrating how the polarization of light changes in response to variations in the magnetic field. The precise measurement of these angles provides insight into the relationship between magnetic field strength and the resulting optical rotation, an effect crucial for applications in magneto-optical devices and materials science.

In this part of study, we investigated the effect of magnetic fields on the polarization of light, which results in a perceptible rotation of the axis of the light spot in the captured images. The process began by capturing light spots under different magnetic field strengths. These images went through a series of image processing steps, including binarization and edge detection to isolate and highlight the edges of the light spots. This allowed us to precisely determine the orientation of the spots' axes.

To measure the axis rotation, each light spot was approximated by fitting a straight line of the form

$$y = mx + cy \tag{8}$$

to its principal axis using a least-squares method. The slope m of each fitted line was then used to calculate the orientation angle with respect to the x-axis via

$$\theta = \arctan(m) \tag{9}$$

By comparing the orientations of two axes, the bisector (median axis) between them was determined. Finally, the angle between this median axis and the x-axis was calculated, providing a quantitative measure of the rotation induced by the magnetic field.

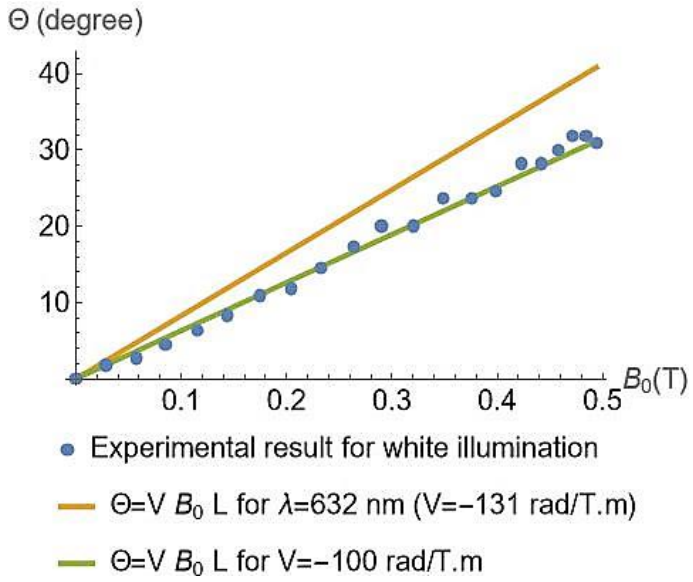


Figure 9. Relationship between the applied magnetic field strength (B_0) and the rotation angle (θ) of the light spot axis, demonstrating the Faraday effect in TGG.

In figure 9 we investigate the relationship between the applied magnetic field strength ($\boldsymbol{B_0}$) and the rotation angle ($\boldsymbol{\theta}$) of the light spot axis, demonstrating the Faraday effect in TGG. Experimental results obtained under white-light illumination (blue dots) are compared with theoretical predictions (solid lines) calculated using Verdet constants of V=−131 rad/(T·m) at λ = 632 nm (orange) and V=−100 rad/(T·m) (green) The close agreement with the V=−100 rad/(T·m)V = -100 \, \mathrm{rad/(T·m)}V=−100rad/(T·m) line indicates that the dominant contribution comes from the green spectral region of the white light, where the effective Verdet constant better matches the experimental behavior. In figure 9, the blue dots represent experimental results under white illumination. The orange and green lines indicate theoretical predictions for different Verdet constants, showing the expected rotation angles for specific wavelengths. The comparison highlights the correlation between the experimental data and the theoretical models.

The results depicted in figure 9 demonstrate the relationship between the magnetic field strength and the rotation angle of the light spot axis, as influenced by the Faraday effect. The experimental data, represented by blue dots, align closely with the theoretical predictions shown by the orange and green lines. This alignment validates the accuracy of our experimental setup and the effectiveness of our image processing techniques in quantifying the rotation angles. The consistency between the experimental results and theoretical models underscores the reliability of our approach in studying the interaction between magnetic fields and polarized light, particularly when using optical vortex and radial polarization, as well as under various illumination conditions including sunlight.

The technological significance of these findings lies in the precise measurement and analysis of the Faraday rotation effect, especially in the context of optical vortex beams and radial polarization under sunlight illumination. These specialized forms of light have unique properties that make them highly suitable for advanced optical applications. Understanding the Faraday rotation in such beams is crucial for the development of next-generation magneto-optical devices, such as optical isolators and circulators, which are essential components in optical communication systems. These devices rely on the controlled rotation of polarized light to manage the directionality of light signals, preventing feedback that could disrupt the operation of lasers and other sensitive equipment. Our method provides a robust framework for characterizing materials' Verdet constants, which are fundamental to designing efficient magneto-optical components using advanced light forms, even in natural light conditions like sunlight.

This study introduces a novel approach to quantifying Faraday rotation by combining advanced image processing techniques with experimental observations of optical vortex beams and radial polarization under sunlight. By using Mathematica's powerful image processing capabilities, including binarization, edge detection, and highlighting, we achieved high precision in measuring the rotation angles. This methodology offers a significant improvement over traditional measurement techniques, which often rely on manual observations and less precise instrumentation. The ability to automate and accurately analyze the rotational effects of magnetic fields on specialized forms of polarized light represents a substantial advancement in optical research. Moreover, the inclusion of sunlight as an illumination source in our experiments demonstrates the practical applicability of our findings in real-world scenarios.

Furthermore, the strong correlation between our experimental data and theoretical predictions highlights the potential for this method to be widely adopted in both academic research and industrial applications. The ability to accurately measure and predict the behavior of optical vortex beams and radially polarized light in magnetic fields can lead to improved material characterization and enhanced performance of magneto-optical devices. This alignment not only confirms the validity of our approach but also paves the way for further innovations in the field, ultimately contributing to the advancement of optical technologies and their applications in communication, sensing, and beyond. The inclusion of optical vortex and radial polarization in our study underlines the cutting-edge nature of our research, positioning it at the forefront of optical science and engineering, while the use of sunlight illumination ensures the relevance of our findings for practical, everyday applications.

In this study, we successfully demonstrated the quantification of Faraday rotation using advanced image processing techniques on light spots influenced by magnetic fields. By employing optical vortex beams and radial polarization, as well as considering various illumination conditions including sunlight, we provided a comprehensive analysis of the rotational effects caused by magnetic fields. Our experimental results showed a strong correlation with theoretical predictions, validating the precision and reliability of our methodology. This experiment not only demonstrated the effectiveness of combining optical components like polarizers and vortex retarders with sophisticated software tools like Mathematica but also provided insights into the behavior of magneto-optic materials under varying magnetic fields. Our findings underscore the potential for using similar setups in future explorations of magneto-optic phenomena and their applications in optical devices.

The ability to accurately measure and analyze Faraday rotation is of significant technological importance, particularly for the development of next-generation magneto-optical devices such as optical isolators and circulators. These components are crucial for ensuring the proper functioning of optical communication systems by managing light signal directionality and preventing disruptive feedback. Additionally, our findings have implications for fiber and cable technologies, guided-wave components, and optoelectronic systems, where precise control and measurement of light polarization are essential.

Our innovative approach combines the use of Mathematica's powerful image processing functions—binarization, edge detection, and highlighting—with experimental observations, allowing for high-precision measurements that surpass traditional techniques. The inclusion of sunlight as an illumination source further underscores the practical applicability of our findings, making them relevant for real-world scenarios. This method can be extended to field trials and new applications, enhancing the deployment and performance of optical networks and switching systems.

Overall, this research paves the way for further advancements in optical science and engineering, offering a robust framework for the characterization of materials and the enhancement of magneto-optical device performance. The integration of optical vortex and radial polarization in our analysis highlights the cutting-edge nature of our work, setting a new standard for studies in this field. Our results contribute to the broader understanding and development of fiber optics, integrated optics, and active and passive guided-wave components, aligning with the scope of journals focusing on both theoretical and experimental advancements in these areas.

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

Artificial Intelligence (AI) Contribution Statement

The authors declare that this manuscript was written and analyzed by the authors. Language editing assistance was used; however, the scientific content, data analysis, and figures are entirely the authors' own work.

Contributions of the Authors

The experiments were performed by M. Tekin, and image processing was carried out by F. Tambağ. Conceptualization and methodology were developed and refined by K. Köksal. All authors contributed to the analysis and writing of the manuscript.

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